1. Suppose I have an instrument that measures the mRNA transcript abundance of a certain gene, and I have a drug that I suspect will alter the expression of that gene when the drug is injected into a rat. I randomly divide a group of eight rats into two groups of four. Each rat in group one is injected with the drug, and each rat in the other group is injected with a control substance. I use my instrument to measure the gene expression in each rat and obtain the following results:

Control group expression: 9, 12, 14, 17. Drug group expression: 18, 21, 23, 26.

(a) Use a two-sample t-test (assuming the within-group standard deviations are the same) to test for a treatment effect.

(b) Use the Wilcoxon rank sum test to perform a non-parametric test on these data. Do the conclusions agree with the above parametric test?

(c) A permutation test would give you the identical p-value as the Wilcoxon rank sum test - why?
   Hint: how many possible group assignments exist for the above data in a permutation test?

2. You want to detect differences in protein abundances between affected probands and normal controls, measured by ELISA. Since you are interested in fold changes, you record your measurements as $\log_2(abundance)$. Thus, a two-fold change in abundance corresponds to a difference ($\Delta$) of 1 on the $\log_2$ scale, a four-fold change corresponds to $\Delta=2$, etc. Also assume that in each group the $\log_2(abundances)$ are independent measurements from a Normal distribution.

(a) Assuming you plan on running ELISAs for the same number of cases and controls, how many subjects do you need to have 80% power to detect a two-fold change in abundance between cases and controls ($\Delta=1$), if the within-group standard deviation of the $\log_2(abundances)$ is the same as the differences in means of the $\log_2(abundances)$, i.e. $\sigma = \Delta$?

(b) How much power do you have if you only ran ten ELISAs per group?

(c) How much power do you have if you only ran ten ELISAs per group, but knew that (in truth) the protein your are interested could not be down-regulated in the affected probands?

3. The following data show blood glucose levels in mg/kg in eighteen rabbits immediately before and two hours after the administration of an analgesic compound. Investigate the effect of analgesia on blood glucose level by applying the sign test, the Wilcoxon signed rank test, and a t-test.

Before treatment: 158, 119, 122, 89, 111, 135, 138, 122, 127, 127, 137, 120, 118, 126, 134, 134, 125, 124
After treatment: 206, 134, 204, 105, 96, 171, 212, 134, 177, 136, 136, 117, 127, 140, 153, 147, 131, 131

The data are also available here.
4. (a) After treatment, 2 out of 20 cells respond to a certain antigen. Calculate a 95% confidence interval for the probability $p$ of response.

(b) In a second experiment, 0 out of 20 cells respond to another certain antigen. Test the null hypothesis that the response rate is 10% versus the alternative that the response rate is less than 10%.

5. Consider the table below containing data from 611 subjects from the Baltimore Longitudinal Study of Aging. The gait speed of these subjects was measured and categorized as either low or normal. These subjects have been further divided by age less than 68 years and older than 68 years.

<table>
<thead>
<tr>
<th></th>
<th>Less than 68</th>
<th>68 and older</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Gait Speed</td>
<td>198</td>
<td>104</td>
</tr>
<tr>
<td>Normal Gait Speed</td>
<td>91</td>
<td>218</td>
</tr>
</tbody>
</table>

(a) Using a Normal approximation based on the formula for large $n$ and medium $p$ in the lecture slides, calculate a 95% confidence interval for the proportion of people with low gait speed among the subjects less than 68, and among the subjects 68 or older.

(b) Calculate a 95% confidence interval for the difference in proportions.

6. Assume you have data for a two-sample t-test, but you consider analyzing the data as paired data instead. Run a simulation study to find out what the consequences for type I error and power are.

(a) Take ten random draws from a standard Gaussian distribution for each group. Analyze the data as paired data, and record the t-statistic and the p-value. Repeat 10,000 times, and make histograms of the test statistics and the p-values. Is the type I error rate affected by the incorrect analysis?

(b) Repeat the above, but with a difference in the populations means of one with-group standard deviation. How much power do you have to detect the difference?

(c) Repeat (b) but now using the proper two-sample t-test. How much power do you have now to detect the difference?