Homework Assignment 10
Solutions

1. (a) As expected, older children have on average larger FEV than younger children. The relationship between age and FEV appears to be linear until about age 12, and then seems to flatten out a bit. This is even more obvious when plotting the logarithm of the FEV measurements. Also, the variability in the FEV measurements seems to increase with age.

(b) See the code.

(c) The least squares estimates for intercept and slope are -0.068 and 0.049, respectively. The respective 95% confidence intervals are (-0.102, -0.034) and (0.045, 0.052). With the logarithm transformation of the FEV measurements, the diagnostic plots look OK, we do not detect any obvious violations of the model assumptions. The increase in log\textsubscript{10} FEV with age is statistically significant. Comparing two children between the ages of 5 and 12 who differ by one year in age, we expect the difference in log\textsubscript{10} FEV to be 0.049, i.e. we expect to see (roughly) a 12% increase in FEV.

[ 3 points ]

2. (a) Please see the code.

(b) The estimated power for the design with 5 biological replicates in each treatment arm and two technical replicates per biological unit is 28%, while the power for the design with 10 biological replicates in each treatment arm and no technical replicates is 56%.

(c) We used 10,000 iterations for each of the power estimates, and the confidence intervals are (0.267;0.285) and (0.548;0.567) respectively.

(d) We have larger power for the design with more biological replicates as we allocate more degrees of freedom to estimate the treatment effects (10 + 10 – 2 = 18 degrees of freedom for a t-distribution). In a nested ANOVA we would test the group mean squares over the subgroup mean squares, with 1 and 2 × (5 – 1) = 8 degrees of freedom respectively (corresponding to a t-distribution with 8 degrees of freedom). With more biological replicates we have a more precise estimate of the group means.

[ 5 points ]

3. Please see the code. The Pearson correlation between height and weight derived from all crew members is 0.985. That seems rather large. Indeed, this estimate is mostly driven by crew member Martin Sauer who is much shorter and lighter than the others. He isn’t a rower, he is the coxswain. Among rowers, the Pearson correlation between height and weight is 0.77.

[ 2 points ]

4. Since \( X \) is non-negative (and at least some of the design points have to be non-zero) the average of all design points is positive, and therefore the correlation \( -\bar{x}/\sqrt{\bar{x}^2 + SXX/n} \) between \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \) is negative.

[ 1 points ]