1. Please see the code for details. If 'abnormal' glucose levels are outside of 2 standard deviations of the mean (i.e. 2 standard deviations above or below the mean), then approximately 4.6% of individuals would have 'abnormal' glucose levels. The 'normal' range of glucose levels is 64 mg/dL to 116 mg/dL. [2 points]

2. (a) Let \( Z = (Y - 30)/5 \). Then \( E(Z) = 0 \) and \( SD(Z) = 1 \).
   (b) Let \( X = -Y \). Then \( E(X) = -E(Y) = -30 \), and \( SD(X) = SD(Y) = 5 \).
   (c) Let \( R = 5 + Y/3 \). Then \( E(R) = 5 + E(Y)/3 = 15 \), and \( SD(R) = SD(Y)/3 = 5/3 \). [2 points]

3. Suppose \( X \sim \text{Normal}(\text{mean}=4,\text{SD}=2) \). Let \( Z = (X - 4)/2 \).
   (a) \( Pr(X = 4) = 0 \) (\( X \) is a continuous random variable).
   (b) \( Pr(X < 6) = Pr(Z < (6 - 4)/2) = Pr(Z < 1) \approx 84\% \).
   (c) \( Pr(X > 0) = Pr(Z > -4/2) = Pr(Z < 2) \approx 98\% \).
   (d) \( Pr(1 < X < 4) = Pr(X > 1) - 1/2 = Pr(X > -3/2) - 1/2 \approx 43\% \).
   (e) \( Pr(|X - 4| > 1) = Pr(|Z| > 2/3) = 2 \times Pr(Z < -2/3) \approx 62\% \).
Please see the code. [3 points]

4. We have \( \bar{x} = 103, s = 10.67, n = 10 \), and thus, the estimated standard error is \( s/\sqrt{n} = 3.37 \). For the critical value we use \( qt(0.975, 9) \) which is 2.26. Thus, the 95% confidence interval can be approximated by \( 101 \pm 2.26 \times 3.37 = (95.37; 110.63) \). Please also see the code. [3 points]

5. We have \( P(-t_{0.95,24} \leq (\bar{X} - \mu)/(S/\sqrt{n}) \leq t_{0.95,24}) = 0.95 \) where \( t_{0.95,24} = 1.71 \) is the 95th percentile of a t distribution with 24 degrees of freedom. The 90% confidence interval is \( \bar{x} \pm t_{0.95,24} \times s/\sqrt{n} = 7 \pm 1.71 \times 3/\sqrt{25} = (5.97; 8.03) \). Please see the code. [2 points]

6. The sampling distribution is normal with mean 3000 ml, and standard deviation \( 400/\sqrt{n} \) (that is, the standard error of the mean). When the sample size \( n \) is 15, the probability that the sample mean will be within \( \pm 100 \) ml of the population mean is 0.67. When the sample size \( n \) is 60, the probability that the
sample mean will be within $\pm 100$ ml of the population mean is 0.95 (please see the code). In general, the larger the sample size, the smaller the standard error, and the larger the probability that the sample mean will be within a given distance of the population mean.

[3 points]

7. Please see the code.

[2 points]

8. The confidence interval would be $\bar{x} \pm 1.96 \times \sigma/\sqrt{n}$. This interval has width $2 \times 1.96 \times \sigma/\sqrt{n}$. Hence we need to find $n$ such that

$$2 \times 1.96 \times 1.5/\sqrt{n} \leq 1 \iff n \geq (2 \times 1.96 \times 1.5)^2 = 34.6.$$ 

Thus we need to measure at least 35 samples. We should get a more precise measuring device.

[3 points]