1. (a) We obtain a p-value of 7.2%, and conclude that the proportion $p$ could reasonably be $1/2$.

(b) The 95% confidence interval for $p$ is (0.22, 0.51).

(c) The 95% confidence interval for $p$ is (0.78, 1.00). A good rule of thumb to obtain the lower limit of the confidence interval is $1 - (3/15) = 0.8$.

[ 3 points ]

2. The test statistic for the $\chi^2$ goodness-of-fit test is 7.70, and the p-value is 0.17. Thus, the data are not inconsistent with the postulated Mendelian model.

[ 3 points ]

3. (a) The chi-square test yields a test statistic of 7.25, and a p-value of 0.40.

(b) The likelihood ratio test yields a test statistic of 7.24, and also a p-value of 0.40.

(c) Fisher’s exact test gives a p-value of 0.43.

(d) As the p-values are large, we fail to reject the null hypothesis. There is no evidence for a difference, among the conditions, in the frequency with which the ticks choose the treated tube. Notice that the asymptotic tests are a bit iffy in principle, as some of the expected number of counts for the untreated tubes are below 5.

[ 3 points ]

4. (a) The expected mean is $E(\hat{p}) = p = 0.3$,

and the variance is $\text{Var}(\hat{p}) = \frac{p \times (1 - p)}{n} = \frac{0.3 \times (1 - 0.3)}{100} = 0.0021$.

(b) See the code.

(c) I get a sample mean of 0.300045, and a sample variance of 0.002101, so very close to the theoretical values above.

(d) See the code. The distribution of the proportions observed in our simulation seem to be close to a Normal distribution. A good rule of thumb is that if $n \times p \times (1 - p) > 5$, the normal approximation holds. Here, $n \times p \times (1 - p) = 21$.

5. Please see the code.
(a) The observations are integers, but the distribution looks reasonably Gaussian when I look at the histogram of the data. Testing for the mean being 0 or not, we get a p-value of 1.1e-13. We reject the null hypothesis that the population mean equals 0.

(b) The sample means of the vital capacity for the two groups are 3.95 and 5.00 respectively. We find this to be a statistically significant difference, the p-value is 0.008. A 99% confidence interval is (−2.06;−0.02).

The result of this comparison may be misleading because age likely has an effect on vital capacity, and the age distributions between the groups are dramatically different: they do not even overlap! The youngest person in the first group is six years older than the oldest person in the other group.

(c) Using the Wilcoxon signed rank test for the react data set, I get a p-value of 2.1e-13, virtually the same as the one derived from the t-test. Using the Wilcoxon rank sum test to assess the hypothesis that the vital capacity is the same in the two groups, I get a p-value of 0.018. This is very close to the p-value I got from the two-sample t-test, however, this would not be considered significant if we control the type I error rate (the significance level) at 1%.