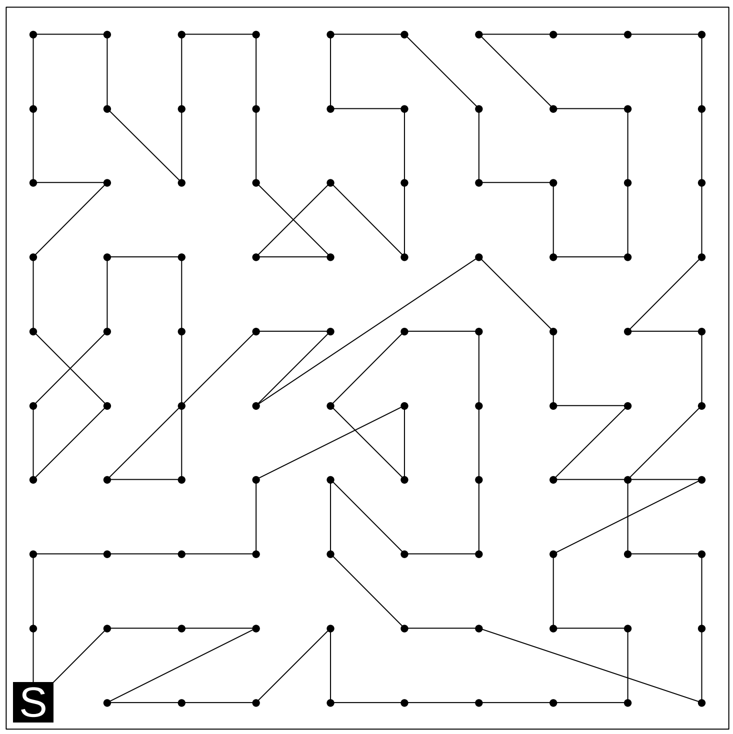
Simulated Annealing

- Simulated annealing is a probabilistic search algorithm.
- The terminology is borrowed from the physics literature.
- For the simulated annealing algorithm we need
  - a scoring function,
  - a move set,
  - a selection probability,
  - an acceptance function.

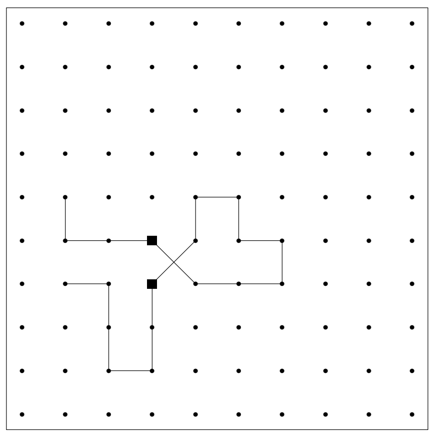
The Traveling Salesman Problem



Which is the shortest path that connects all cities?

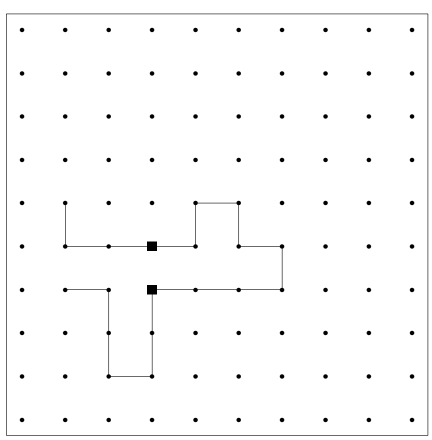
The Traveling Salesman Problem (cont.)

$\cdots$ 43 44 35 36 37 47 46 56 55 45 34 24 14 13 $\cdots$



$\cdots$ 43 44 45 55 56 46 47 37 36 35 34 24 14 13 $\cdots$

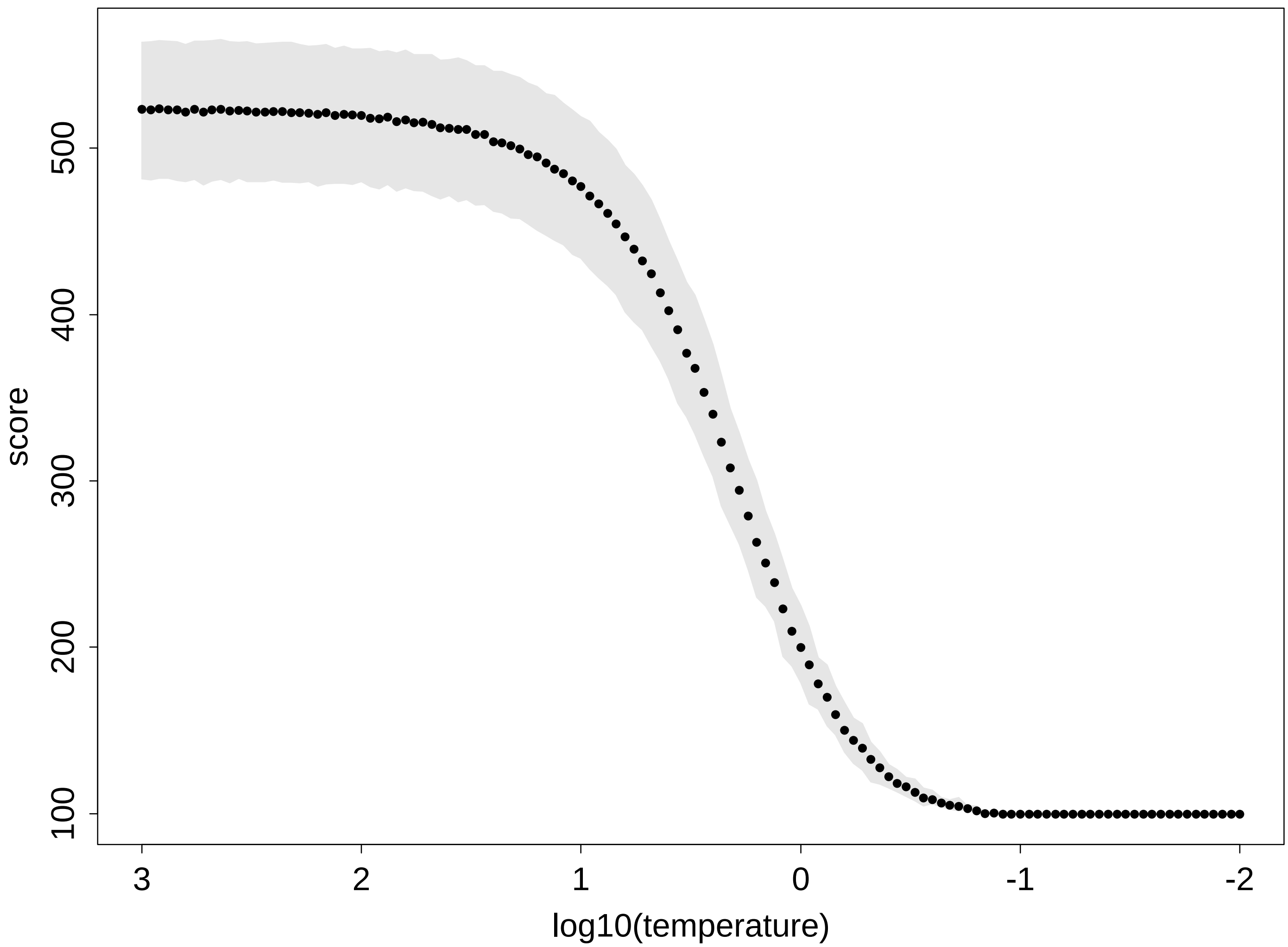
↑ ↑



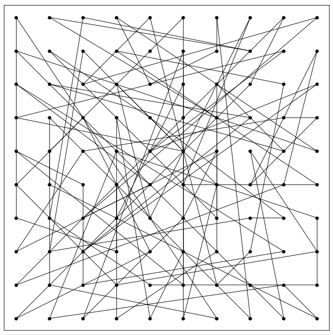
Properties of our Simulated Annealing run

- We use the acceptance function $\alpha(\epsilon_{\text{old}}, \epsilon_{\text{new}}, t) = \min\left\{1, \exp\left(\left[\epsilon_{\text{old}} - \epsilon_{\text{new}}\right]/t\right)\right\}$
- We run homogeneous Markov chains at constant temperatures.
- We constructed the move set to be irreducible and aperiodic, therefore each homogeneous Markov chain has a limiting distribution $\pi_t(S)$.
- The limit (as $t \to 0$) of those distributions assigns probability 1 to the optimal scoring states.
- With limited resources, we cannot guarantee to find an optimal scoring state.

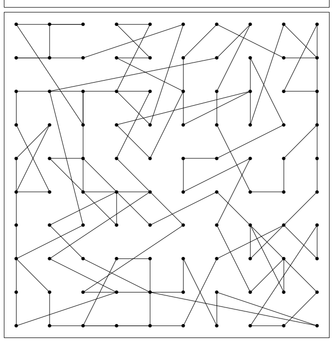
The Traveling Salesman Problem (cont.)



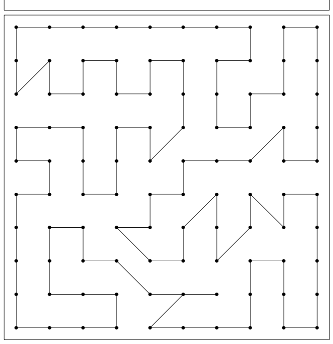
The Traveling Salesman Problem (cont.)



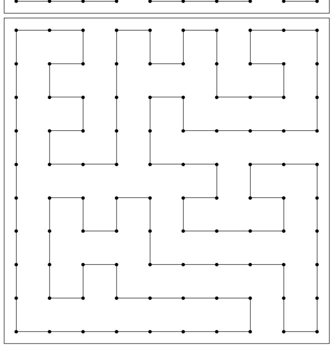
The initial path.



The tour after 60 steps.

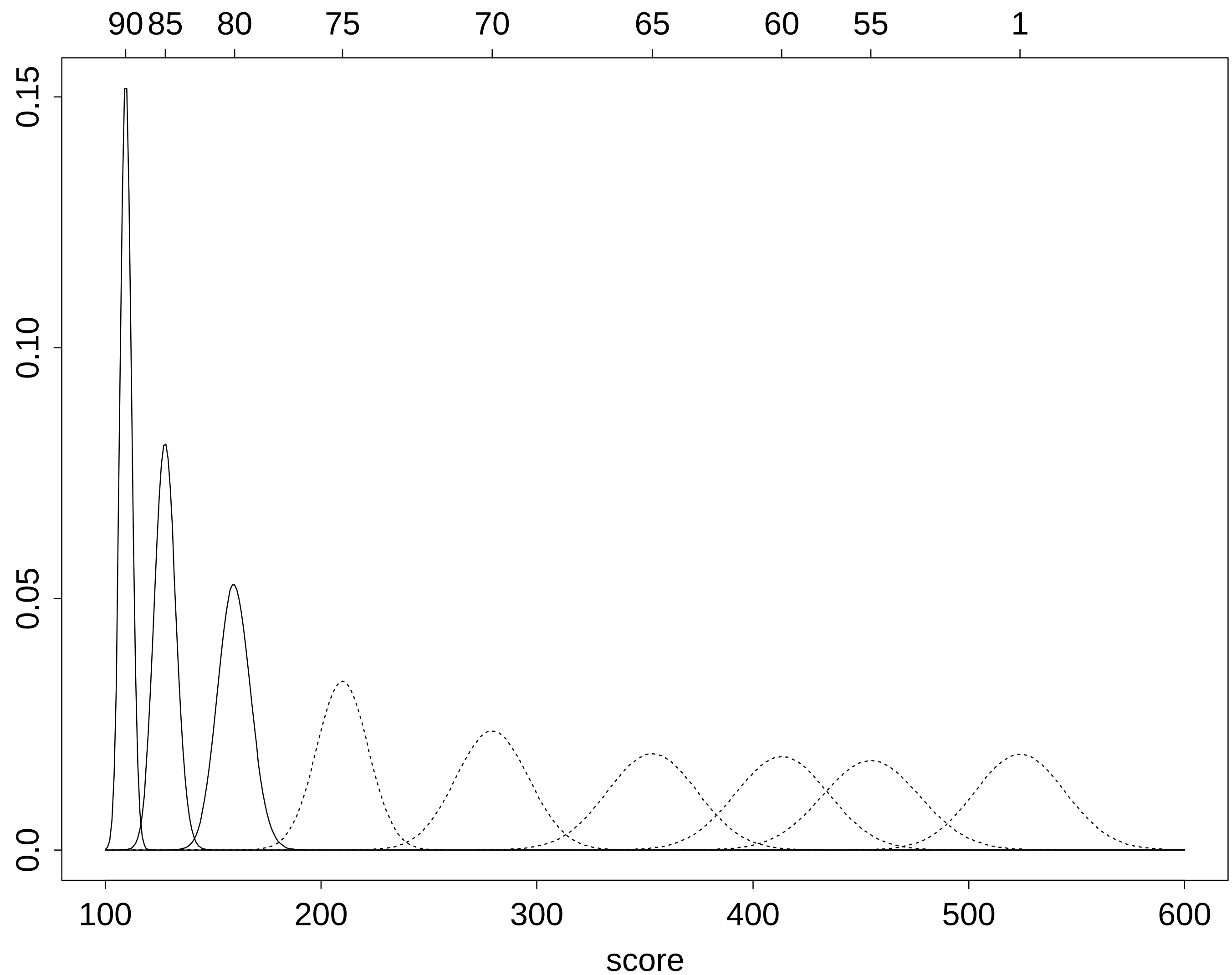


The tour after 90 steps.



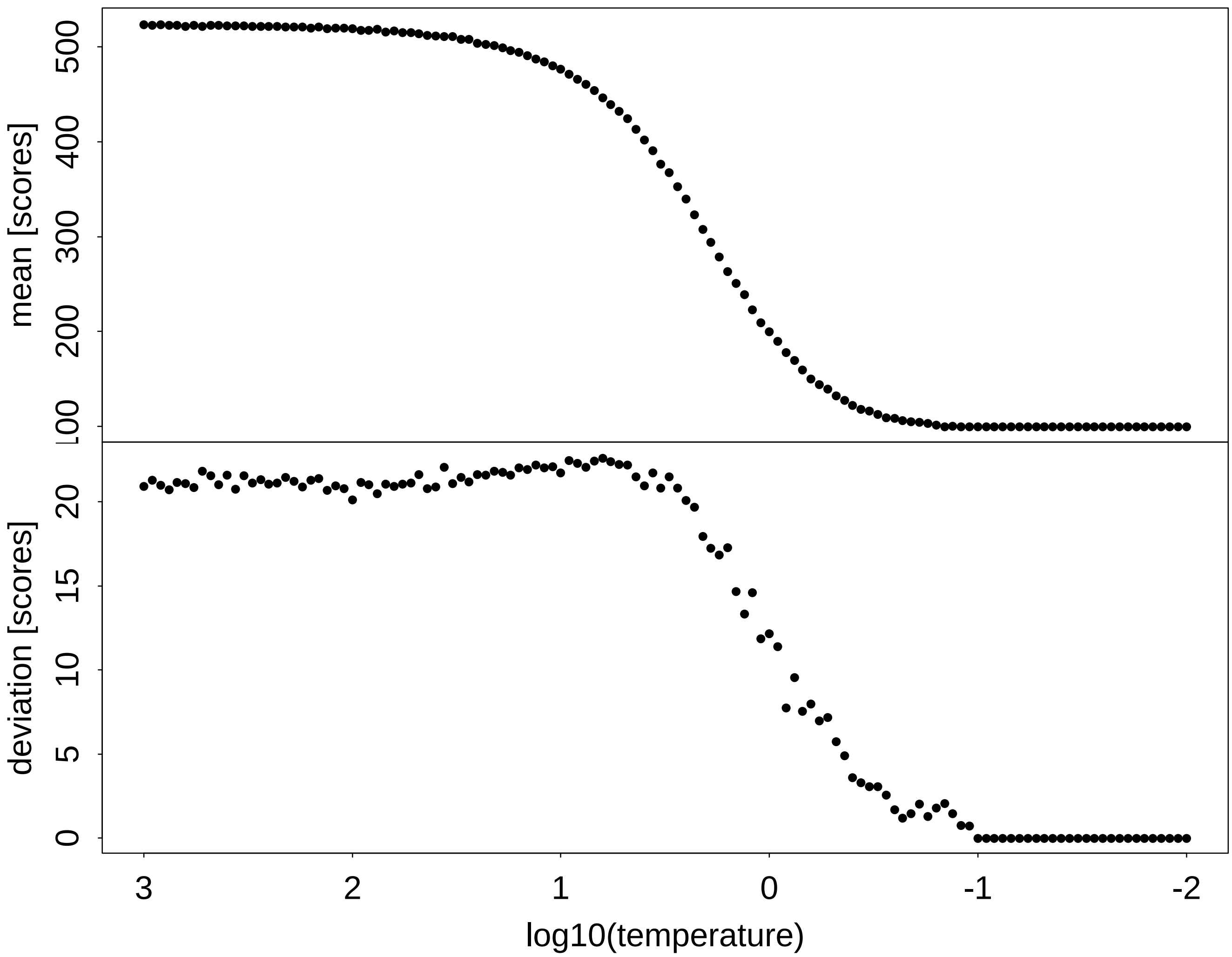
The tour after 125 steps.

The Traveling Salesman Problem (cont.)



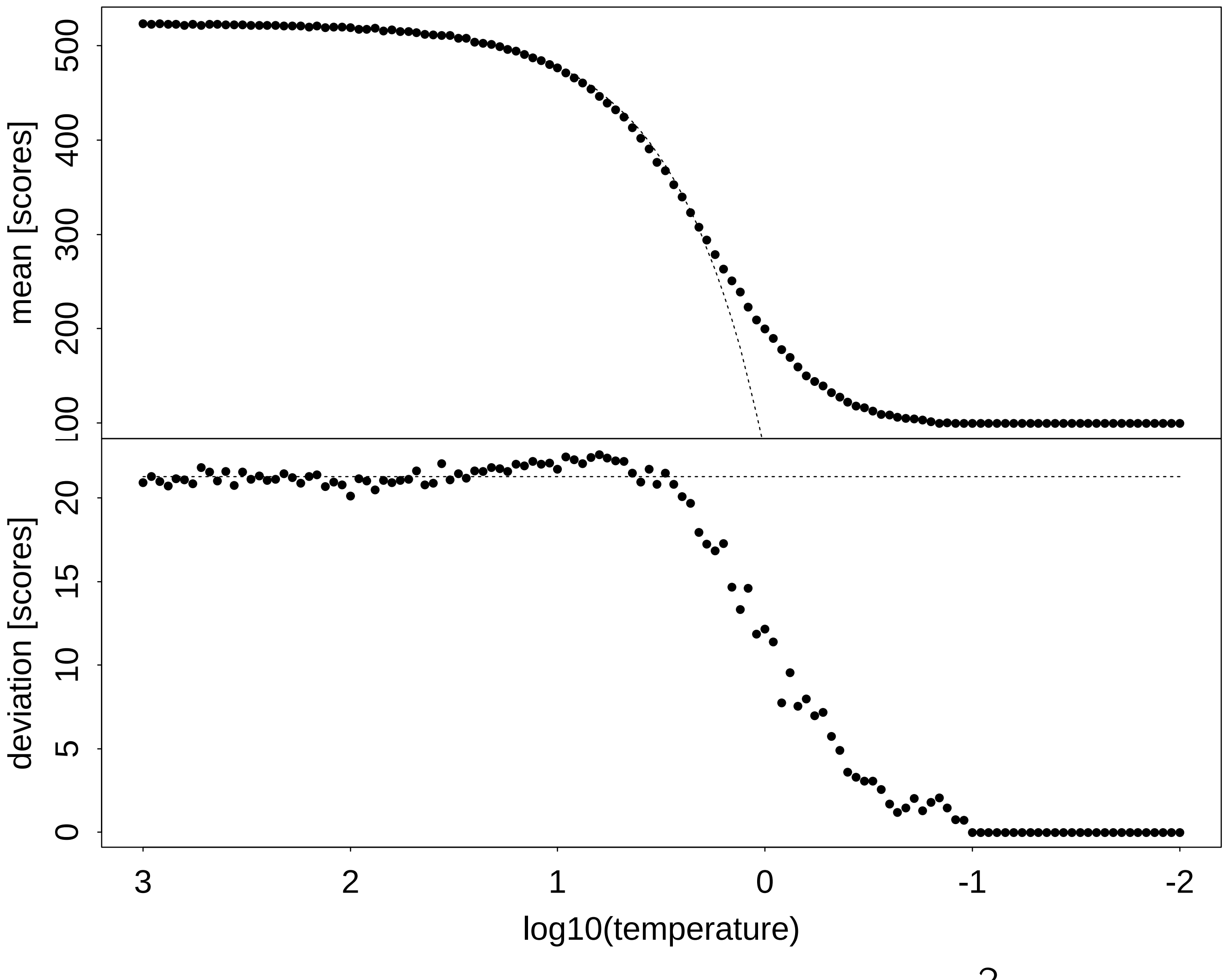
Anything noteworthy in the score distributions?

The Traveling Salesman Problem (cont.)



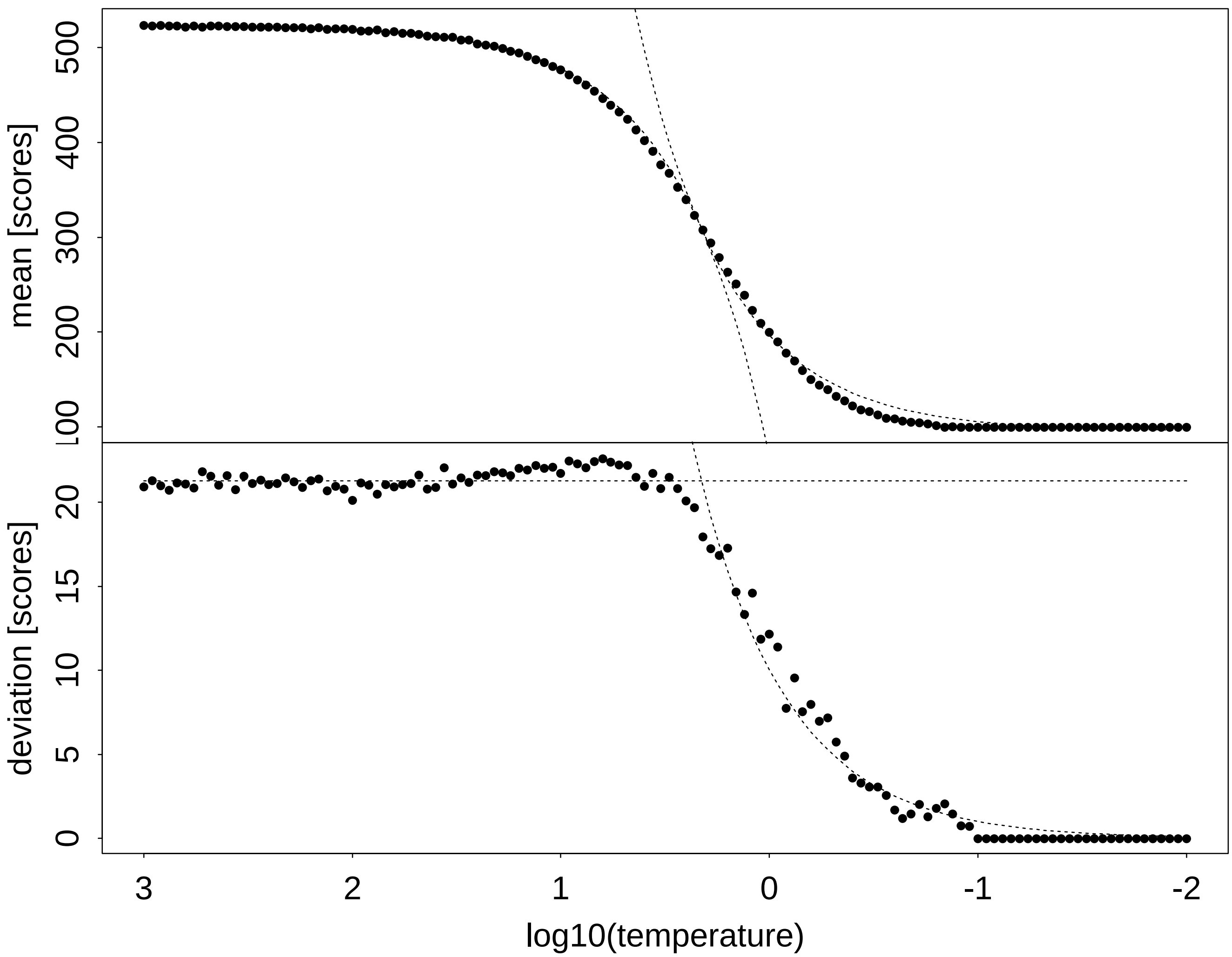
The mean and the standard deviation of the scores.

The Traveling Salesman Problem (cont.)



Normal scores with mean $\mu(t) = \mu_\infty - \frac{\sigma_\infty^2}{t}$ and standard deviation $\sigma(t) = \sigma_\infty$.

The Traveling Salesman Problem (cont.)



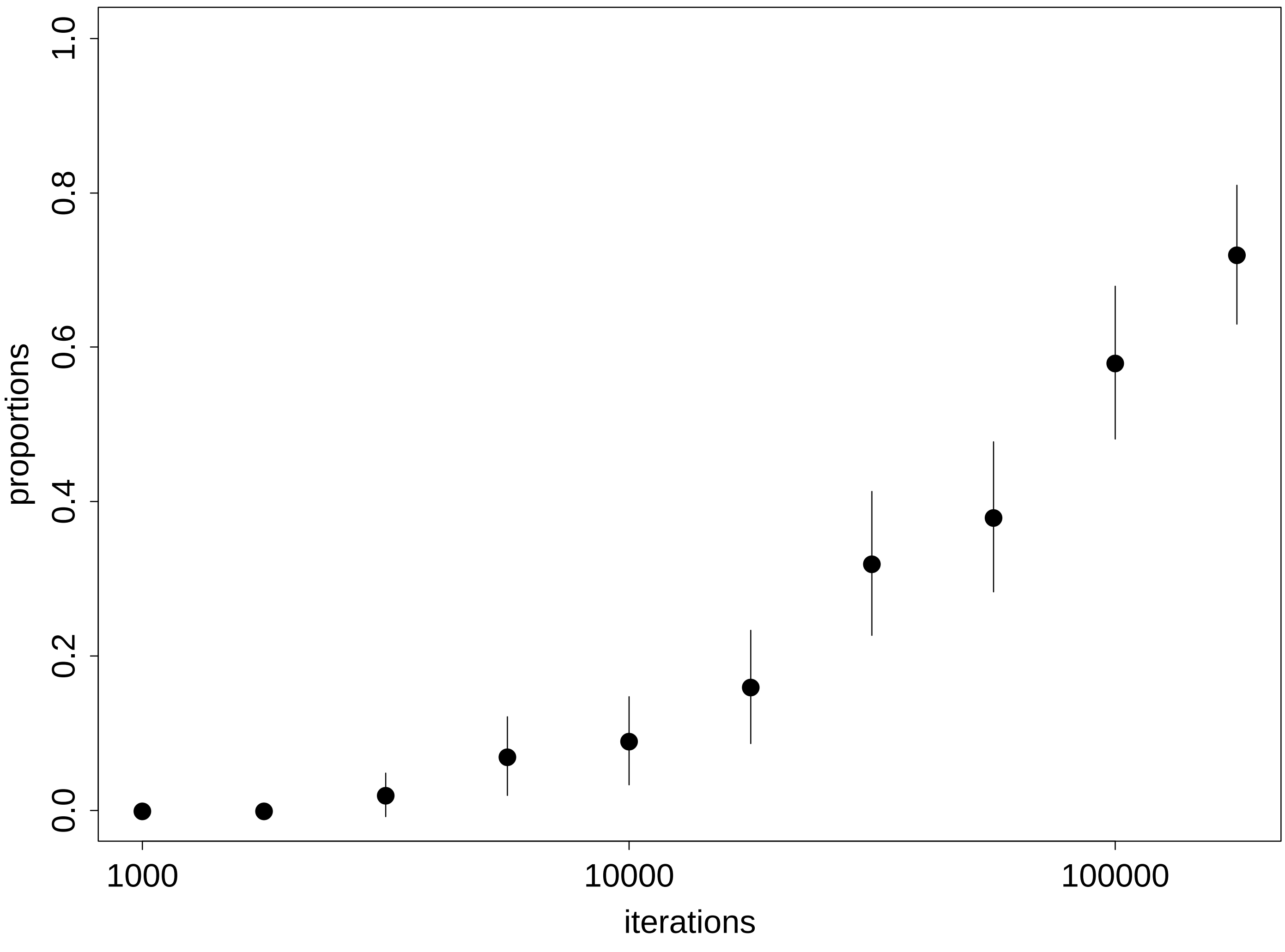
Gamma scores with mean $\mu(t) = \mu_\infty - \frac{\sigma_\infty^2}{T}\left(2 - \frac{t}{T}\right)$ and standard deviation $\sigma(t) = \sigma_\infty \frac{t}{T}$ added.

The Traveling Salesman Problem (cont.)

There is a region of weak and a region of strong control! Let μ_∞ be the mean and σ_∞ be the standard deviation at temperature $t = \infty$.

	$t \geq T$	$t_e < t \leq T$
$\mu(t)$	$\mu_\infty - \frac{\sigma_\infty^2}{t}$	$\mu_\infty - \frac{\sigma_\infty^2}{T}\left(2 - \frac{t}{T}\right)$
$\sigma(t)$	σ_∞	$\sigma_\infty \frac{t}{T}$

The Traveling Salesman Problem (cont.)



Iterations versus success rate on a slimmed down version of the algorithm.