Handout

Using the notation from class, assume that the confidence intervals for μ_A and μ_B do not overlap. Let k be the approximated degrees of freedom. Without loss of generality, assume $\bar{x} > \bar{y}$. It follows that

$$
\begin{aligned}
\bar{x} - t_{n-1} \times \frac{s_A}{\sqrt{n}} \; > \; \bar{y} + t_{m-1} \times \frac{s_B}{\sqrt{m}} \quad &\Longleftrightarrow \quad \bar{x} - \bar{y} - t_{n-1} \times \frac{s_A}{\sqrt{n}} - t_{m-1} \times \frac{s_B}{\sqrt{m}} \; > \; 0 \\
&\Longrightarrow \quad \bar{x} - \bar{y} - t_k \times \left(\frac{s_A}{\sqrt{n}} + \frac{s_B}{\sqrt{m}} \right) \; > \; 0 \\
&\Longleftrightarrow \quad \bar{x} - \bar{y} - t_k \times \sqrt{\left(\frac{s_A}{\sqrt{n}} + \frac{s_B}{\sqrt{m}} \right)^2} \; > \; 0 \\
&\Longleftrightarrow \quad \bar{x} - \bar{y} - t_k \times \sqrt{\frac{s_A^2}{n} + \frac{s_B^2}{m} + \frac{2 s_A s_B}{\sqrt{nm}}} \; > \; 0 \\
&\Longrightarrow \quad \bar{x} - \bar{y} - t_k \times \sqrt{\frac{s_A^2}{n} + \frac{s_B^2}{m}} \; > \; 0
\end{aligned}
$$

Hence the confidence interval for $\mu_A - \mu_B$ does not cover zero.