Handout

Assume that you have three independendent measurements X_1, X_2, X_3, with $\text{var}(X_i) = \sigma^2$.

It follows that $\text{var}(\bar{X}) = \sigma^2/3$.

Assume that you have to subtract a baseline B from each measurement. What is $\text{var}(X_i - B)$?

If B is just a constant, then $\text{var}(X_i - B) = \text{var}(X_i) = \sigma^2$.

If B is a measurement with $\text{var}(B) = \sigma_B^2$, then $\text{var}(X_i - B) = \text{var}(X_i) + \text{var}(B) = \sigma^2 + \sigma_B^2$.

What is the variance of the average of those values, i.e. what is the variance of $\sum_i (X_i - B)/3$ .. ?

$$\text{var}\left(\sum_i (X_i - B)/3\right) = \text{var}\left(\frac{1}{3}\left(X_1 + X_2 + X_3 - 3 \times B\right)\right) = \text{var}\left(\bar{X} - B\right) = \text{var}(\bar{X}) + \text{var}(B) = \frac{\sigma^2}{3} + \sigma_B^2.$$

If each experiment has its own baseline B_i, measured independently, with $\text{var}(B_i) = \sigma_B^2$, then

$$\text{var}\left(\sum_i (X_i - B_i)/3\right) = \text{var}\left(\bar{X} - \bar{B}\right) = \text{var}(\bar{X}) + \text{var}(\bar{B}) = \frac{\sigma^2}{3} + \frac{\sigma_B^2}{3} = \frac{\sigma^2 + \sigma_B^2}{3}$$