Practice Problems (Confidence Intervals for Proportions)

- In a sample of patients from a certain population, 9 out of 10 individuals respond to the treatment.

  1. Show that (0.5550 ; 0.9975) is a 95% confidence interval for the probability of response in that population. subject to rounding error
  2. Given the data, is there evidence against the assumption that the true response probability is 50% in the population?
  3. What would the 95% confidence interval be if we had observed 10 responders among the 10 subjects sampled?

  Solution:

  1. We have

     $P(X \geq 9|p = 0.5550) =$

     $P(X = 9|p = 0.5550) + P(X = 10|p = 0.5550) =$

     $\binom{10}{9} \times 0.5550^9 \times 0.4450^1 + \binom{10}{10} \times 0.5550^{10} \times 0.4450^0 =$

     $0.0222 + 0.0028 = 0.025,$

     and

     $P(X \leq 9|p = 0.9975) = 1 - P(X = 10|p = 0.9975) = 1 - 0.9975^{10} = 0.025.$

     Therefore, (0.5550 ; 0.9975) is a 95% confidence interval for the probability of response in the population.
  2. There is evidence against the assumption that the true response probability in the population is 50%, as 0.5 is not contained in the 95% confidence interval.
  3. If we had observed 10 responders out of 10 subjects sampled, the 95% confidence interval would have been $(0.025^{1/10}; 1) = (0.69; 1)$. Note that the rule of thumb gives $1 - 3/10 = 0.7$ for the lower bound.