Nerve cells contain branched structures called dendritic trees. In a sample of 36 nerve cells from the brains of newborn guinea pigs, investigators counted the number of branch segments per cell. The sorted data is shown here with a histogram.

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> branches

 [1] 20 21 21 23 23 24 26 27 27 27 28 28 29 29 29 30 30 30 31 33 33 34 35 35 35
[26] 37 37 40 41 43 48 49 51 51 54 57

> hist(branches)
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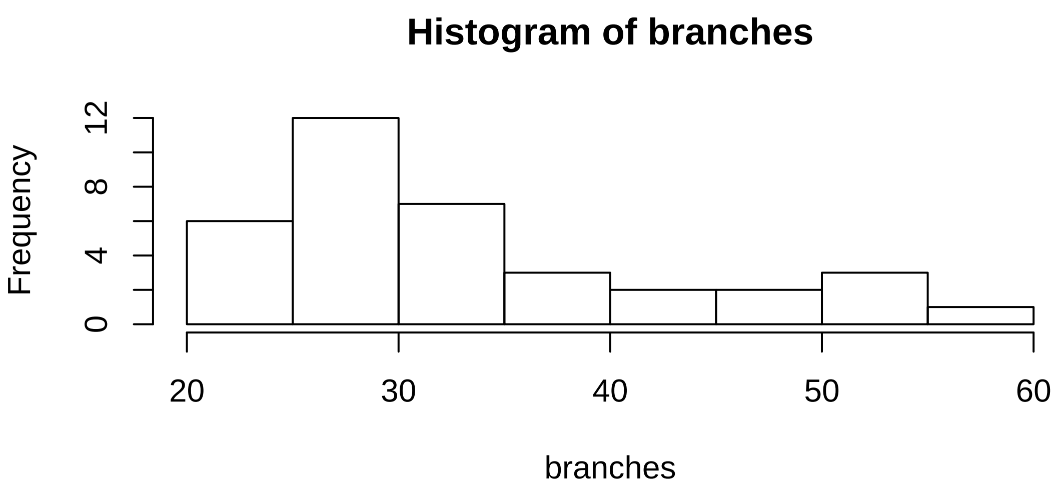

(a) Use your calculator to find the mean and standard deviation of the sample data.

Solution: $\bar{y} = 33.8$ and $s = 9.9$.

(b) Assume the the data is a random sample of nerve cells from a population of newborn guinea pigs. Find a 98% confidence interval for the population mean number of branch segments.

Solution: $33.8 \pm 2.438 \times 1.65$, or 33.8 ± 4.

(c) Interpret the confidence interval in part (b) in the context of the problem.

Solution: We are 98% confident that the mean number of branch segments in all nerve cells of newborn guinea pigs is between 29.8 and 37.8.

(d) Circle the true statements below.

1. The confidence interval constructed in part (b) is not valid, because the histogram shows that the sample is skewed.
2. The confidence interval in part (b) is approximately valid despite the skewness apparent in the sample because the central limit theorem implies that the sampling distribution of the sample mean will be approximately normal for large enough samples.
3. If the sampled branch counts were not independently sampled, but rather there were several nerve cells taken from each of a few newborn guinea pigs, then the confidence interval in part (b) may not be valid.

Solution: Statement 2. is, in fact, a better explanation than Statement 1, but you may not have yet developed the intuition to see that the skewness in the histogram is not so severe and that a sample size of $n = 36$ is sufficient for the normal approximation to be accurate. Statement 3. is true.

(e) Assume that the population mean and standard deviation are 35 and 10 branch segments respectively. What is the probability that the sample mean in a new random sample of size 36 would be smaller than 33?

Solution: $\Pr\{\bar{Y} < 33\} = \Pr\{Z < -1.2\} = 0.1151$.

(f) Explain why the calculation in part (e) is valid even if the population is not approximately normal.

Solution: The sample size is sufficiently large (for the observed skewness) so that the sampling distribution of the sample mean will be normal, so the calculation is valid.