In a cross of two heterozygotes in each of two traits, the Mendelian expected ratio of offspring types are 9:3:3:1 for the possible phenotypes (DD,DR,RD,RR) where D stands for the dominant phenotype, R stands for the recessive phenotype, and we have given the two traits an arbitrary order of first and second. In an experiment, the number of offspring of each type is: 128 DD, 39 DR, 31 RD, and 14 RR.

(a) Carry out a χ^2 goodness-of-fit test to test if the observed counts are consistent with the Mendelian expected counts. State hypotheses, compute a test statistic, and use the table to find a range for the p-value.

Solution:
H_0: The probabilities of the phenotypes are 9/16, 3/16, 3/16, and 1/16.
H_A: The probabilities of the phenotypes are something else.
There are a total of 212 individuals and the expected counts are this sum times 9/16, 3/16, 3/16, and 1/16, or 119.25, 39.75, 39.75, and 13.25.
The χ^2 test statistic is 2.62.
From the table, the p-value is larger than 0.2.

(b) How strong is the evidence against the Mendelian hypothesis for these traits?

Solution: There is very little evidence against the null hypothesis. The observed data is consistent with the Mendelian theory.

(c) Assume that the Mendelian expected ratios are correct and that offspring types are independent of one another. What is the probability that in a future experiment with 32 offspring that there would be one or fewer offspring of type RR?

Solution: The number of offspring of type RR fits the binomial model — binary outcomes (RR or not), independent trials, fixed sample size, and same probability of success for each trial — with $n = 32$ and $p = 1/16$.
The probability of one or fewer successes is

$$\begin{aligned} \Pr\{Y \leq 1\} &= {}_{32}C_0(1/16)^0(15/16)^{32} + {}_{32}C_1(1/16)^1(15/16)^{31} \\ &= (1/16)^0(15/16)^{32} + 32(1/16)^1(15/16)^{31} \\ &= 0.1268 + 0.2705 = 0.3973 \end{aligned}$$

For each remaining part, circle TRUE or FALSE. If the answer is FALSE, either explain why it is false or make a small change to make the statement true.

(d) TRUE or FALSE:

A bucket contains 100 balls, 10 of which are black and 90 of which are white. The balls are mixed thoroughly and a random sample of 16 balls is taken (without replacement). The number of black balls in the sample is a binomial random variable.

Solution: False. The trials are not independent, as the probabilities of future trials depend on the results of previous draws. For example, there is probability 0 of drawing 16 black balls.

(e) TRUE or FALSE:

Assume that a population is approximately normal and that we are able to take random samples from this population. There is approximately a 95% chance that a 95% confidence interval for a population mean μ will contain μ.

Solution: True. That's why we are 95% confident that it does.

(f) TRUE or FALSE:

Assume that a population is approximately normal and that we are able to take random samples from this population. A sample of size 100 yields a 95% confidence interval for μ of 2.12 ± 0.32 To achieve a 95% confidence interval that has a margin of error of 0.16 in a subsequent sample from a similar population, we should double the sample size.

Solution: False. The margin of error is $t \times s/\sqrt{n}$. When the sample size increases, t and s won't change much. You need a sample four times as large to halve the margin of error.