

Please recall from the syllabus: You are expected to type your homework. Each solution should include a brief summary of the problem asked. You may write in by hand mathematical notation, graphs, and so on. You may make minor corrections and additions by hand as well. Please include the problem number (as listed on the assignment) in your write-up. Some assignment problems consist of two or more problems from the book. Please use a stapler or paper clip rather than tearing the corners of multiple papers together. Please include your name and the discussion section you attend (M 2:25, M 4:00, T 1:20, T 4:00) at the top of your assignment to make it easier for us to return your homework to you. (The discussion section you attend might not be the one you for which you are registered, which is okay.) The best time to turn in homework is in class (or before). You may turn it in to Professor Larget's office by 2pm on the due date. Future assignments that are not typed neatly (as described above) or are incomplete will not receive full credit.

1. Problems 2.38 and 2.39.

Solution: Data is CK levels (measured in U/l) from 36 healthy men. The sample mean and standard deviation are 98.3 and 40.4 respectively. The interval $\bar{y} \pm (1 \times s)$ is (57.9,138.7) which contains 26 of 36 values, or 72%. The interval $\bar{y} \pm (2 \times s)$ is (17.5,179.1) which contains 34 of 36 values, or 94%. The interval $\bar{y} \pm (3 \times s)$ is (-22.9,219.5) which contains 36 of 36 values, or 100%.

The empirical rule predicts 68%, 95% and more than 99% which is fairly close to what we observe.

2. Problem 3.6

Solution: The data counts the number of broods of starling by size (which ranges from 1 to 10) for a total of 5000 broods. A brood is chosen at random. Y is the size of the brood.

$$(a) \Pr\{Y = 3\} = 610/5000 \doteq 0.122.$$

$$(b) \Pr\{Y \geq 7\} = (130 + 26 + 3 + 1)/5000 \doteq 0.032.$$

$$(c) \Pr\{4 \leq Y \leq 6\} = (1400 + 1760 + 750)/5000 \doteq 0.782.$$

3. Problem 3.7

Solution: In the previous problem, the 5000 broods contain a total of 22,435 young. Suppose one of the young is chosen at random and that Y' is the size of the brood of that young.

$$(a) \Pr\{Y' = 3\} = (3 \times 610)/22435 \doteq 0.082. \text{ (There are } 3 \times 610 \text{ young in the 610 broods of size 3.)}$$

$$(b) \Pr\{Y' \geq 7\} = ((7 \times 130) + (8 \times 26) + (9 \times 3) + (10 \times 1))/22435 \doteq 0.051.$$

(c) If a brood is selected by choosing a young at random, larger broods will be more likely to be chosen than smaller ones because they have more young. That is why the probability of selecting a young from a brood of size 7 or higher is greater than the probability of selecting a brood of size 7.

4. Problem 3.10

Solution: This problem deals with early pregnancy tests and their accuracy.

(a)

$$\begin{aligned} \Pr\{+\} &= \Pr\{+|\text{pregnant}\} \Pr\{\text{pregnant}\} + \Pr\{+|\text{not pregnant}\} \Pr\{\text{not pregnant}\} \\ &= 0.98 \times 0.10 + 0.01 \times 0.90 \\ &= 0.107 \end{aligned}$$

(b)

$$\begin{aligned} \Pr\{+\} &= \Pr\{+|\text{pregnant}\} \Pr\{\text{pregnant}\} + \Pr\{+|\text{not pregnant}\} \Pr\{\text{not pregnant}\} \\ &= 0.98 \times 0.05 + 0.01 \times 0.95 \\ &= 0.0585 \end{aligned}$$

5. Problem 3.11

Solution: This problem follows up on the setting of the previous problem.

(a)

$$\begin{aligned}\Pr\{\text{pregnant}|+\} &= \Pr\{+|\text{pregnant}\} \Pr\{\text{pregnant}\} / \Pr\{+\} \\ &= (0.98 \times 0.10) / 0.107 \\ &\doteq 0.916\end{aligned}$$

(b)

$$\begin{aligned}\Pr\{\text{pregnant}|+\} &= \Pr\{+|\text{pregnant}\} \Pr\{\text{pregnant}\} / \Pr\{+\} \\ &= (0.98 \times 0.05) / 0.0585 \\ &\doteq 0.838\end{aligned}$$

6. Problem 3.13

Solution: The density curve shows the distribution of lengths (in μm) of a parasite.

(a) $\Pr\{20 < Y < 30\} = 0.41 + 0.21 = 0.62$.

(b) $\Pr\{Y > 20\} = 0.41 + 0.21 + 0.03 = 0.65$.

(c) $\Pr\{Y < 20\} = 0.01 + 0.34 = 0.35$.

7. Problem 3.14

Solution: This problem uses the settings of the previous problem and considers a sample of two parasites.

(a) The probability that both parasite lengths are less than 20 is $(0.35)^2 = 0.1225$.

(b) The probability that the first parasite length is shorter than 20 and the second is greater than 25 is $0.35 \times 0.24 = 0.084$.

(c) The probability that one parasite length is shorter than 20 and the other is greater than 25 is $2 \times 0.35 \times 0.24 = 0.168$ because there are two different ways this can occur.